The basic knowledge necessary to solve an equation with separated variables.

One of the fundamental types of differential equations is an equation with separated variables. The process of solving this type of equation involves separating the terms containing y and x, moving them to different sides of the equation, and then applying integration. After integrating, its important not to forget to add the constant of integration to one side. Below is an example problem illustrating the process of solving this type of equation.

We consider the following differential equation:

$$y + 2x^2y' = 0 \tag{1}$$

Where:

$$y(x) \tag{2}$$

Is our encountered function.

Separation of variables

If possible, we perform the commonly known separation of variables x and y. By separating variables, we mean separating terms containing the variable x from those containing the variable y. To accomplish this separation, we can assign these terms to functions $h(x)$ and $g(x)$, as shown below:

$$g(x) = \frac{1}{2x^2} \tag{3}$$

$$h(x) = y \tag{4}$$

Apart from these elements, our equation also contains the term y', which can also be written as:

$$y' = \frac{dy}{dx} \tag{5}$$

After substituting this expression, it can be observed that in subsequent steps, multiplying the equation on both sides by dx enables integrating both sides with respect to different variables.

Integration of separated variables

$$\frac{dy}{dx} = \frac{y}{2x^2} \tag{6}$$

To do this, we multiply both sides by dx and move $y(x)$ to the opposite side of the equation:

$$\frac{dy}{y} = \frac{1}{2x^2}dx \tag{7}$$

Then, to "go back" to the original function and obtain the result in the form of the f y, we integrate both sides of the obtained equation. Importantly, when integrating, it's essential to remember to add the constant of integration in any form, typically denoted as C or D.

$$\int \frac{1}{y}\,dy = \int \frac{1}{2x^2}\,dx \tag{8}$$

After integration, we obtain the following result:

$$\log(y) = -\frac{1}{2x} + D \tag{9}$$

Simplification of the solution obtained after integrating the equations

After rearrangment a solution has the following form:

$$y(x) = C_1 e^{-\frac{1}{2x}} \tag{10}$$

The obtained expression represents the solution to a differential equation with separated variables. The constant of integration can be determined when initial conditions are provided in the problem, i.e., the value of the function for a specific x.